DEPARTMENT OF MATHEMATICS

Syllabus
Master of Philosophy Programme in Mathematics

2011
DEPARTMENT OF MATHEMATICS

Master of Philosophy Programme in Mathematics

Course Overview

The Master of Philosophy Program in Mathematics is being offered based on a credit system similar to the other programmes offered by the University. The M. Phil. program has two semesters with each semester spreading through 15 weeks. The candidate has to submit the dissertation within two months after the second semester ends. It can be extended by two more months on specific request. Those who fail to submit the dissertation within the extended time period can register for a further extension of four more months with a payment of 50% of the course fee. There will be only two repeat chances (within three years after registration) for the course work papers and no revaluation of papers at any stage of the program. The time taken from the admission till the submission of the dissertation shall be considered as the duration of the M. Phil. Program.

This programme is aimed at developing students into mature researchers and preparing them for higher research degree and teaching.

Department Goal:

To provide all students with training in mathematics that will serve as part of foundation for research and teaching.

Course Content

The course content for the first semester is

<table>
<thead>
<tr>
<th>Marks</th>
<th>credits</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4</td>
<td>60 hrs</td>
</tr>
<tr>
<td>1) General Research Methodology 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Specific Research Methodology 100</td>
<td></td>
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</tbody>
</table>

The course content for the second semester is

<table>
<thead>
<tr>
<th>Marks</th>
<th>credits</th>
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</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4</td>
<td>60 hrs</td>
</tr>
<tr>
<td>1) Paper 1 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Paper 2 100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dissertation & Viva-Voce 200

The Dissertation work includes presentation on the project proposal (50 Marks), double valuation of the final dissertation (100 marks), and viva-voce examination (50 Marks).
Assessment of course work

Each paper of the semester will be assessed upon 100 marks (Continuous Internal Assessment (CIA) – 45 marks, Attendance –5 marks and End Semester Examination - 50 marks).

The internal assessment (comprising various components such as seminar, literature survey, presentation, class test and so on) should be done periodically and the CIA marks should be sent to the HOD and a copy forwarded to the General Research Coordinator as per the following guidelines:

- **CIA 1**: 10 Marks assessment before the 2\textsuperscript{nd} Month
- **CIA 2**: 10 Marks assessment before the 3\textsuperscript{rd} Month
- **CIA 3**: 25 Marks assessment before the 4\textsuperscript{th} Month

The HOD will hand over the consolidated CIA marks, before the End Semester Examination to the Controller of Examinations, in the format as per the requirement of the office of Examinations.

Students who fail to complete CIA requirements on the specified date may be given another chance to repeat the CIA, before the next CIA, at the discretion of the teacher and with the consent of the Coordinator.

At the end of each semester there shall be an end semester examination for each paper/elective. The design/pattern of the questions and question papers need not be the same for all disciplines. However, the design/pattern shall be approved by the Deans. The Maximum marks for each end semester examination will be 100 and the duration is three hours.

Two sets of independent question papers for each subject, completely sealed, should be sent to the COE through the Coordinator. The question paper should reach the coordinator at least 15 days in advance.

There is no minimum mark required for CIA. The minimum mark to pass in ESE for each paper is 50%. The minimum mark to pass in each paper is 50% aggregate of CIA marks and ESE marks.

In case a candidate fails due to low marks in CIA, he/she can re-register for that subject with a payment of required fee and complete the CIA requirements, by attending the classes along with the other candidates as directed by the coordinator.

If a candidate fails due to low marks in ESE, he/she can appear for the subject by paying the prescribed fee, along with the other candidates as scheduled by the office of examinations.

Each candidate shall work under the supervision of a guide. Specific guiding for the research program/Dissertation may commence from the beginning of the II semester. The HOD/Coordinator will allot guides to the candidates by the end of the first semester or in the beginning of second semester depending upon the area of specialization.
Submission of Dissertation

The title page of dissertation, contents etc. should strictly conform to the format as prescribed by the university and the dissertation (all copies) should carry a declaration by the candidate and certificate duly signed and issued by the guide. The dissertation should be hard bound.

The candidates will be granted a maximum period of six months, after completing the course to submit the dissertation.

The M.Phil. dissertation will not be accepted for assessment, unless the candidate has paid the prescribed fees.

The candidate shall submit five hard bound copies and a soft copy (CD) of his/her dissertation work for assessment.

Adjudication of the M. Phil Dissertation

The dissertation submitted by the candidate under the guidance of the guide will be assessed by two experts (one internal and one external).

The candidates also have to appear for final viva-voce. Assessment based on the viva-voce and the dissertation, along with the assessment of theory papers of both I & II semesters will be considered to declare the results.

The candidates will be provided with marks card and a degree certificate. The grade points and the class obtained will be entered along with the marks on the marks card.

Cancellation M.Phil. Admission

The admission of the candidate will be cancelled under the following circumstances.

1) Fails to secure 85% attendance.
2) Fails to submit the documents/ requirements related to internal assessment.
3) Does not pay the course fee within the stipulated time.
4) Fails to submit the dissertation within the stipulated time.
## Course structure

### I Semester

<table>
<thead>
<tr>
<th>Paper Code</th>
<th>Subjects</th>
<th>Hrs. / week</th>
<th>Marks</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMT 131</td>
<td>1. General Research Methodology</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 132</td>
<td>2. Mathematical Analysis</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>8</strong></td>
<td><strong>200</strong></td>
<td><strong>8</strong></td>
</tr>
</tbody>
</table>

### II Semester

(Electives: Choose any one – each elective has two papers)

<table>
<thead>
<tr>
<th>Paper Code</th>
<th>Subjects</th>
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</thead>
<tbody>
<tr>
<td>RMT 131</td>
<td>1. General Research Methodology</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 132</td>
<td>2. Mathematical Analysis</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
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<th>Hrs. / week</th>
<th>Marks</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMT 231a</td>
<td>1. Differential Equations and Computational Methods</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 232a</td>
<td>2. Advanced Fluid Mechanics</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 231b</td>
<td>1. Submanifolds of Riemannian Manifolds</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 232b</td>
<td>2. Riemannian Geometry</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 231c</td>
<td>1. Computational Graph Theory</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>RMT 232c</td>
<td>2. Advanced Graph Theory</td>
<td>4</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>8</strong></td>
<td><strong>200</strong></td>
<td><strong>8</strong></td>
</tr>
</tbody>
</table>
## DISSERTATION

<table>
<thead>
<tr>
<th>Components</th>
<th>Marks</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation on the research proposal</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>Double valuation of the dissertation</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>Viva-Voce examination</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td>8</td>
</tr>
</tbody>
</table>
Modular Objective:

SEMESTER I: (General papers)

In the first semester, students are offered two theory papers, viz, General Research Methodology and Specific Research Methodology (Mathematical analysis). The modular objectives of these papers are:

RMT 131: General Research Methodology

The research methodology module is intended to assist students in planning and carrying out research projects. The students are exposed to the principles, procedures and techniques of implementing a research project.

RMT 132: Mathematical Analysis

The objective of this paper is to help students understand the principal concepts such as convergence and divergence of improper integrals, Lebesgue Measure, Lebegue integral and multivariate Calculus.

SEMESTER II: (Electives)

In the second semester, three elective papers are included. Students are supposed to choose any one of the three electives. Each elective has two papers. The modular objectives of each elective paper are:

Elective 1. Fluid Mechanics

Paper 1:
RMT 231(a): Differential Equations and Computational Methods

This course will focus on advanced concepts in both ordinary and partial differential equations. Special emphasis is given to the computational methods.

Paper 2:
RMT 232(a): Advanced Fluid Mechanics

This paper provides a opportunity to explore various instability problems in fluid mechanics such as Rayleigh-Benard instability, Marangoni instability, Double Diffusive convection, convection in porous media and convective instability in non-Newtonian fluids.
Elective 2. Riemannian Geometry

Paper1:
RMT 231(b): Submanifolds of Riemannian Manifolds

This paper focuses on the study of the submanifolds of Riemannian manifolds and hence on hypersurfaces and various other types of submanifolds.

Paper2:
RMT 232(b): Riemannian Geometry

This paper focuses on differentiable manifolds and hence on Riemannian manifolds, a central concept which plays an important role in several different mathematical and/or scientific specialized areas.

Elective 3. Graph Theory

Paper1:
RMT 231(c): Computational Graph Theory

This paper concerns the principal concepts of graph theory such as enumeration problems, networks, spectral graphs and graph complexity. This paper could be a useful tool for technical research in the area of Graph Theory.

Paper2:
RMT 232(c): Advanced Graph Theory

This paper covers advanced topics in graph theory such as optimization of vertex and edge coloring, matching algorithm, four color conjecture, domination theory and so forth. This paper provides the foundations for advanced research in Graph Theory.

DISSERTATION

Major emphasis is given to the dissertation work on a chosen research problem. The modular objective includes research proposal, presentations on the research work done, submission of dissertation and viva-voce examination. The publications of the research work in refereed journals and presentation of research work in national/international conferences/symposia/seminars will be encouraged.
Course Curriculum

Semester - I (General papers)

Paper 1: RMT 131
GENERAL RESEARCH METHODOLOGY

Unit I 15 Hours


Unit II 15 Hours


Unit III 15 Hours


Unit IV 15 Hours

Originality in research, resources for research, Research skills, Time management, Role of supervisor and Scholar, Interaction with subject expert, The Computer: Its Role in Research, Case study interpretation: minimum 5 case studies

Texts and References:


Horaine Blaxter, Christina Hughes, Malcolm Tight *How to research*, Viva Books Pvt Ltd, 1999

Bell, J., *Doing your research project*, Viva, 1999 (NIAS)


Gilham, B., *Case study research methods*, Continuum 2005 (NIAS)


Gregory, I., *Ethics in research*, Continuum, 2005 (NIAS)


Morgan, D. L., *Focus groups as qualitative research*, Sage Pub., 1988 (NIAS)

Illingham, Jo., *Giving presentations*, OUP, 2003 (NIAS)

Denscombe, M., *The good research guide*, Viva, 1999 (NIAS)

Blaxter, L., *How to research*, Viva, 2002 (NIAS)

Ezzy, D., *Qualitative analysis*, Routledge, 2002 (NIAS)

Patton, M. Q., *Qualitative evaluation and research methods*, Sage Pub, 1990 (NIAS)

Unit I - Improper integrals 10 Hours

Convergence of improper integrals of the first and second kind, absolute and conditional convergence, integral test for series, Abel’s test, Dirichlet’s test, Cauchy Principal value.

Unit II – Abstract integration and positive Borel measures 20 Hours

Concept of measurability, simple functions, elementary properties of measures, integration of positive functions, integration of complex functions, the role played by sets of measure zero, the Riesz representation theorem, regularity properties of Borel measures, Lebesgue measure, continuity properties of measurable functions.

Unit III – $L^p$ spaces 10 Hours

Convex functions and inequalities, the $L^p$ – spaces, approximation by continuous functions.

Unit IV – Functions of several variables 20 Hours

Linear transformation, continuity, differentiability, continuously differentiable functions, inverse function theorem, implicit function theorem.

Texts and References:

Semester – II (Electives)

Elective I: **FLUID MECHANICS**

**Paper 1: RMT 231(a)**

**DIFFERENTIAL EQUATIONS AND COMPUTATIONAL METHODS**

**Unit I:** 15 Hours


**Unit II:** 15 Hours


**Unit III:** 15 Hours


**Unit IV:** 15 Hours

**Introduction to Mathematica Software:**

**Numerical Computation:** Numerical solution of differential equations, numerical solution of initial and boundary value problems, numerical integration, numerical differentiation, matrix manipulations, optimization techniques.

**Graphics:** Two- and three-dimensional plots, parametric plots, typesetting capabilities for labels and text in plots, direct control of final graphics size, resolution etc.
**Texts and References:**


**Paper 2 : RMT 232(a)**

**ADVANCED FLUID MECHANICS**

**Unit I:**  
**Shear Instability:** Stability of flow between parallel shear flows - Squire’s theorem for viscous and inviscid theory – Rayleigh stability equation – Derivation of Orr-Sommerfeld equation assuming that the basic flow is strictly parallel.

**Unit II:**  

**Unit III:**  
**Porous Media:** Different models to study convection problems in porous media – Normal mode analysis – Principle of exchange of stabilities – first variation principle – Solution for free-free boundaries – Double Diffusive convection in porous media.

**Unit IV:**  

**Texts and References :**


Research Papers:


Elective II: RIEMANNIAN GEOMETRY

Paper 1: RMT 231(b)

RIEMANNIAN GEOMETRY

Unit I 15 Hours
DIFFERENTIABLE MANIFOLDS: Introduction to Manifolds – The space of tangent vectors at a point of $\mathbb{R}^n$ – related theorems – Vector fields on open subsets of $\mathbb{R}^n$ – Inverse function theorem – Definition of Differentiable manifold – Examples: Euclidean plane – Tangent space at a point on a manifold - Vector field on a manifold – Lie algebra of the vector fields on a manifold – Frobenius theorem.

Unit II 15 Hours
TENSOR FIELDS ON MANIFOLDS: Tensor fields – Differential Forms and Lie Differentiation – Covariant Derivative of Tensors – Connections – Torsion Tensor – Symmetric Connections – Curvature tensor field

Unit III 15 Hours

Unit IV 15 Hours
CONTACT MANIFOLDS: Contact Structures – Contact structures in $\mathbb{R}_{2n+1}$ $\text{T}_3$ and $\mathbb{R}_{n+1}$ $\times \text{PR}_n$ – Almost Contact Structures – Contact metric structures – Contact Metric structures in $\mathbb{R}_{2n+1}$ - Normal Almost Contact Structures - K-contact Structures - Sasakian manifolds – Sasakian Structures on $\mathbb{R}_{2n+1}$

Texts and References:


SUBMANIFOLDS OF RIEMANNIAN MANIFOLDS

Unit I

SUBMANIFOLDS OF RIEMANNIAN MANIFOLDS: Submanifolds of Riemannian manifolds – Induced connection and second fundamental form – Equations of Gauss, Codazzi and Ricci – Mean curvature and Gaussian curvature.

Unit II

UMBILICAL AND MINIMAL SUBMANIFOLDS: Totally umbilical submanifolds, Totally geodesic submanifolds, Minimal submanifolds in Euclidean space.

Unit III

HYPERSURFACES: Hyper surfaces of a Riemannian manifolds – Hypersurface of \( \mathbb{R}^n \) – Fundamental forms on hypersurface of \( \mathbb{R}^n \).

Unit IV

SUBMANIFOLDS OF CONTACT DISTRIBUTION: Integral submanifolds of the Contact Distribution – Integral submanifolds of odd dimensional spheres \( S^{2n+1}, S^5, S^3 \).

Texts and References:


Elective III : GRAPH THEORY

Paper 1: RMT 231(c)

COMPUTATIONAL GRAPH THEORY

Unit I: 15 Hours


Unit II: 15 Hours


Unit III: 15 Hours

Spectral graphs: Introduction to the laplacian and eigen values. Basic facts about the spectrum of a graph. Eigen values of weighted graphs. Eigen values and its related problems.

Unit IV: 15 Hours

Graph complexity: Basic concepts in complexity theory, relation between P, NP and NP-complete. Basic NP-complete problems: 3-Satisfiability, 3-Dimensional matching, vertex cover and clique, Hamiltonian circuit and partitions.

Texts and References:

1. F. Harary : Graph Theory, Addison -Wesley, 1969
UNIT -I  
15 Hours
Vertex and edge covering, vertex and edge independence number, Gallai theorems in terms of vertex and edge. Matching- perfect matching, augmenting paths, maximum matching, Hall’s theorem for bipartite graphs, the personnel assignment problem, a matching algorithm for bipartite graphs, Chinese postman problem. Factorizations, 1-factorization, 2-factorization.

UNIT –II  
15 Hours
Vertex and edge coloring, the minimization problem for vertex and edge coloring, Brook’s theorem for vertex coloring, Vizing’s theorem for edge coloring, Simple sequential coloring algorithm, Welsh and Powel algorithm, Smallest-last sequential coloring algorithm, color partition, four color conjecture, elementary homomorphism for coloring, homomorphism interpolation theorem, chromatic polynomials.

UNIT-III  
15 Hours

UNIT-IV  
15 Hours
Basic concepts in domination theory, total /connected/ independent domination number, total/connected/ independent domatic number, bounds on total /connected/ independent domination in terms of vertex, edge, diameter, degree, covering, independence and connectivity of a graph, Neighbourhood number and independent neighbourhood number. Bondage number, Cobondage number and Nonbondage number.

Texts and References: